REPORT OF THE ARITHMETIC COMMITTEE.

INTRODUCTORY.

Perhaps no subject in the school curriculum, with a possible exception of English, has received as much attention as arithmetic. There is a wealth of text-books of wide variety on the topic, as well as an abundance of pedagogical and philosophical works. If we add to this, the multitude of articles published in the various journals, we are at once convinced of the diffculty of properly evaluating the results thus far attained and of making correct deductions from them.

During the more recent years, the most popular catch words in business and professional circles have been "efficiency" and "standardization." The latest profession is that of the efficiency engineer. If his activity is focused on a city system or upon a university, he is said to make a *survey*. Less pretentious are numerous investigations and experiments, many of which have involved great expenditures of time and thought, which are increasingly abundant in pedagogical journals.

Taking these facts into consideration, your committee has reached the conclusion that the time and funds at its command will not warrant the committee in inaugurating additional experimentation, nor is the best current opinion so accurately focused as to justify your committee in outlining model courses.

We believe we can render the wisest service to the Association at this time by calling attention to a few of the larger investigations that have recently been made; by reviewing contributions of conspicuous value that have lately appeared in the journals; and by making a few suggestions and recommendations growing out of the work of the committee.

* Report of the Arithmetic Committee adopted by The Association of Teachers of Mathematics in the Middle States and Maryland at the New York Meeting, November 28, 1914.

The Preliminary Report.

In an earlier report published in the *Mathematics Teacher,* Vol. V, No. 3, your committee gave abstracts of the replies of twenty-five leaders of educational thought to two questions:

I. What is the prime reason for teaching arithmetic in Grades V to VIII as our schools are generally organized?

II. What is the greatest need for reform if we are to accomplish this result?

While the answers to these questions were naturally worded somewhat differently, the thought contained was quite uniform:

I. "To acquire and retain speed and accuracy in the fundamental operations taught in the earlier grades and to show how these operations may be applied to problems of daily life, both of civic and local interest."

II. "The substitution of present day material, adjusted to actual business practice, for useless and antiquated problems,—simplification on practical lines."

In most schools, the work of the first four grades comprises the four fundamental operations. In some schools some knowledge of fractions is added to this ground. Courses of study in the later grades were formerly planned on the hypothesis that the elementary operations were completely mastered in the first four grades. Recent investigations in large city systems have shown that this is not the fact. There is a wide divergence in the ability to add, subtract, multiply, and divide in all the upper grades and in the high school. In an eighth grade for instance, may be found children, who as far as arithmetic is concerned, belong to all of the seven grades below.

For this reason, newer courses of study and recent text-books provide for review drill to cultivate speed and accuracy. There has also been great improvement in the content of material used. Problems dealing with modern business, industrial and social environment have largely given place to the worn out problems of the older text-books.

Experimental Studies.

A complete list of experimental studies is beyond the scope of this report. An excellent summary of such tests may be found in Dr. Ernest C. McDougle's "Contribution to Arithmetic."*

A large number of reported investigations is based upon a relatively small number of observations. In some cases, doubtless, deductions thus based will probably hold universally, in other cases they will not.

The investigators who have studied groups of 6,000 or more children are E. O. Lewis, J. M. Rice, C. W. Stone, S. A. Courtis, and Miss Rose A. C. Carrigan, the last named being officially assigned by the Boston School Committee to continue the extensive investigations of the Boston schools made by S. A. Courtis.

Rice's Test.—In 1902, Dr. J. M. Rice gave a test of eight problems to 6,000 children of grades IV to VIII in eighteen schools of various types in seven cities. He reached the conclusion that supervision was the most important factor in city systems, and the success of supervision was ultimately dependent upon the efficiency of the system of examinations employed. He was the first to show that results of school work in arithmetic were not proportional to the time given the subject on the school program.

Stone's Test.—In 1908, Dr. C. W. Stone published the results of six years' study of detailed data from arithmetic tests in twenty-six widely scattered school systems.†

He concludes: "The course of study may be the most important single factor, but it does not produce abilities unless taught. The other essential features for successful teaching are teachers and children of usual abilities, a reasonable time allotment, intelligent supervision and adequate measurement of results by tests."‡

Some other special conclusions of Dr. Stone are as follows:

The same system occupies a decidedly different rank in fundamental operations from that held in reasoning.

* Pedagogical Seminary, June, 1914, Vol. XXI., pp. 194, 195.

† "Arithmetical Abilities and Some Factors Determining Them," by C. W. Stone, Ph.D.

‡ *Ibid.*, p. 91.

The correlation of reasoning is lowest, 32 per cent., with addition; highest, 52 per cent., with subtraction.

Brown's Test.—In 1911 and subsequently, J. C. Brown's application of the Stone standard tests to 6th, 7th, and 8th grade children, showed the value of regular five-minute drills in the fundamentals; permanent improvement resulted in a marked degree also in text-book work.*

Courtis's Tests.—After experimenting with the Stone tests, S. A. Courtis, of Detroit, devised standard tests on a more carefully devised basis than any tests previously constructed. Basing his studies on more than nine thousand children, he was able to classify the combinations of two figures according to difficulty in each of the four fundamental operations. Thus he could construct standard scales of measurement in each of these operations and could duplicate them making several series of equivalent tests. In the preliminary work much difficulty was experienced in controlling individual variation from the accepted standards. The testing of 5,000 children in Detroit and 33,000 in New York for the Hanus Committee on School Inquiry made possible unusual uniformity in the conditions under which the tests were given and in tabulating the results, so that the data were both more extensive and more reliable than from any previous tests. Later the 20,000 children in the Boston schools were tested during the school year 1912–1913. These extensive investigations together with tests in nearly one hundred other school systems in some fifteen states, easily place these tests in the front rank of such investigations.

Mr. Courtis's conclusions from these tests are:

(*a*) That school work as now conducted is exceedingly inefficient.

(*b*) That results vary but little from school to school and from city to city.

(*c*) That the factors, course of study, method of work, ability of the teachers, are of relatively slight importance compared to the basic factor, the differences in the aptitudes and needs of individual children.

(*d*) That the problem of problems in education today (1914)

* *Journal of Educational Psychology,* February, 1911; November and December, 1912.

is the working out of administrative and pedagogic methods that will enable the teacher of a large class to give individual instruction to each particular child according to its needs.

To give some idea of the consistency and the uniformity of the results of these tests the following scores are given:

MEDIAN SCORES, SERIES B.

FEBRUARY, 1914, TABULATION.

Attempts. *Test 1—Addition.* Rights.

Source of Scores.	Attempts.				Rights.			
	Detroit.	Boston.	General.	Probable June Standards.	Detroit.	Boston.	General.	Probable June Standards.
No. in Group.	1,315.	20,441.	3,618.		1,315.	20,441.	3,618.	
Grade 3			3.6	4.0			.7	2.0
4	5.4	5.3	4.7	6.0	2.7	2.6	1.9	3.0
5	6.7	7.2	5.8	7.5	3.9	3.7	3.9	4.0
6	8.4	8.3	8.0	9.0	4.6	4.9	3.7	5.0
7	9.2	8.4	8.2	10.5	5.4	4.6	4.7	6.5
8	10.2	11.0	9.7	12.0	6.7	7.8	5.6	8.0
			Test 2—Subtraction.					
Grade 3			3.8	4.0			9.7	1.0
4	5.6	5.5	5.7	6.0	3.1	2.5	1.2	3.0
5	8.0	7.6	6.5	8.0	5.5	5.6	4.5	5.5
6	8.8	8.8	8.9	10.0	6.2	6.3	6.1	7.0
7	9.8	9.1	10.2	11.5	7.3	6.9	7.8	8.5
8	12.3	11.4	11.7	12.5	9.5	8.6	8.4	10.0
			Test 3—Multiplication.					
Grade 4	3.6	2.2	3.9	4.5	1.0	1.3	1.3	1.5
5	6.4	5.8	6.0	7.0	3.8	3.3	2.6	4.0
6	7.4	6.9	7.2	8.5	4.8	4.8	4.5	5.5
7	9.6	8.0	8.4	10.0	6.0	5.1	5.2	6.5
8	10.5	9.5	9.9	11.5	7.5	7.6	6.4	8.0
			Test 4—Division.					
Grade 4	1.9	2.6	3.1	3.5	.7	.7	.7	1.0
5	4.9	4.5	4.5	5.0	2.7	2.0	2.3	3.0
6	6.4	5.8	5.8	6.5	4.4	3.3	4.3	5.0
7	8.6	6.1	7.6	8.5	7.1	5.1	5.8	7.0
8	10.3	8.8	9.2	10.5	8.8	6.9	6.3	9.0

(Manual of Instruction, The Courtis Standard Tests, page 75, Department of Co-operative Research, 82 Eliot Street, Detroit, Mich.)

The score in the above table of median scores is the *number* of examples attempted in the "Attempts"; and the *number* of examples right in the "Rights." A time limit was set in each test,—in no case over eight minutes. Each test contained sufficient examples to more than occupy the time of the swiftest computor.

To give an idea of the range of difficulty in these standard tests, a unit example from each is given below. The other examples are similar to the ones here given but with other combinations. (It must be borne in mind that the examples are carefully constructed according to the formulas and principles developed by the author, and are not chance combinations of figures.)

SAMPLE EXAMPLES FROM THE TESTS.

Test 1. Addition (Time 8 Min).	Test 2. Subtraction (Time 4 Min).	Test. 3. Multiplication (Time 6 Min).	Test 4. Division (Time 8 Min).
297	75088824	3597	94)85352
925	57406394	73	
473			
983			
315			
661			
794			
177			
124			

While Mr. Courtis has also devised other tests in combining the elementary operations, in copying figures, and in simple reasoning, the four tests of which the scores are given above, have had widest application and are the best basis from which to form deductions. It will be noticed that Mr. Courtis in these tests has wisely studied only the foundations of arithmetic ability. The simple tests in reasoning which he devised, led him to the conclusion that ability in reading was an important factor in solving arithmetical problems in simple reasoning.

Mr. Courtis's tests have clearly demonstrated the complexity of the problem facing every teacher of arithmetic. Though her class is officially designated, Fourth, Fifth, Sixth Grade, etc., she is really teaching pupils belonging (from the arithmetic point of view) to all grades, and to make matters worse, most of

the pupils belong in other grades than the one she is expected to teach. This tremendous flaw in our system of classification is the fundamental source of our ineffective results. In reading Mr. Courtis's conclusions based on these extensive tests (as previously quoted) we must bear in mind this lack of scientific classification, which so hampers the work of a trained and able teacher as to almost reduce her work to the level of the unskilled and indifferent worker. Space will not permit further consideration and criticism of many other of Mr. Courtis's observations. His aim to reach the individual child and his recognition of the wide range of aptitudes and needs of individual children are safe planks in any educational platform, whether it be of the teacher or that of the educational investigator.

Boston Tests Now in Progress.—Further experiments and investigations by means of the Courtis tests are now being conducted in the Boston schools by a special bureau assigned to the work under the direction of Miss Rose A. Carrigan. The results of these investigations are not yet ready for publication, but may be looked for in the forthcoming reports of Superintendent Dyer.

Jessup's Questionnaire.—In the *Elementary School Teacher,* Vol. 14, June, 1914, Prof. Walter Jessup presents the results of a questionnaire on topics to be taught and time to be given to them. This was sent to all cities with a population of 4,000 and over and to one sixth of the county superintendents of the United States.

The returns indicate a strong tendency to omit many of the traditional topics of arithmetic and to emphasize other phases of the subject that are of more immediate social and economic value. The report includes a list of twenty-five topics showing the percentage of superintendents who favor the elimination of each. A list of nineteen topics together with the percentage of superintendents who favor increased emphasis upon each of them gives suggestive and valuable evidence to superintendents, supervisors and teachers of arithmetic.

The report also states the median time spent on arithmetic in each of the grades. The author suggests that this median percentage of recitation time might profitably be adopted as a

standard until scientific investigations have proven it to be in error.

In the November number of the same magazine Prof. Jessup gives additional data, from these same questionnaires, bearing on the proper time of introduction of an arithmetic text. The returns show that while there is a good deal of variation in this respect, 56.1 per cent. of the schools reporting introduce a text in arithmetic in the third grade and 27.7 per cent. of the schools introduce a text in the fourth grade. In a very few schools the text is introduced as early as the first grade, while in some schools no text is used prior to the sixth grade. The greatest variations are found in the smaller cities.

Lewis's Experiment.—The experiment of E. O. Lewis (*Journal of Experimental Pedagogy,* Vol. 2, No. 2, June, 1913) to ascertain the comparative popularity of arithmetic is called by the author but a "first word" on the subject. Eight thousand pupils in the schools of London and South Wales were asked to write their school subjects in order of preference. Mr. Lewis makes his report in such a way that it is impossible to state the relative popularity of arithmetic from the individual viewpoint. The papers from each class examined were averaged, forming class preferences; the class preferences were then compared. Arithmetic appears to hold a middle ground in the esteem of the classes. The author points out the fallacy of applying this position to the individuals and states: "Arithmetic was generally either very high on the list or very low. Generally speaking, arithmetic is popular with a considerable section of the boys, especially in the classes; and disliked by a large section of the girls."

Suggestions and Recommendations.

1. Experimental pedagogy should be encouraged and teachers should investigate in a scientific manner some of the many questions that face our profession in teaching arithmetic.

2. Standards in most of our schools for speed and accuracy in the fundamental operations are now so low that much good will be accomplished by the testing method now generally used by Courtis and others which emphasizes these qualities and makes them the basis of many deductions as to the individual ability of the pupil and the pedagogical skill of the teacher.

3. In conformity with the experience of a great many teachers, experiments have clearly shown the value of continued brief, snappy drill to increase accuracy and speed and to maintain a creditable standard, provided the drill is intelligently directed toward the objects in view. We recommend that such drills form a definite part of the program of study.

4. While we feel that great improvement is desired in speed and accuracy in American schools, yet teachers must always keep in mind that mechanical skill in arithmetic is *but a part* of the complex training which good arithmetic teaching should give. Among the many other qualities desired, the cultivation of the power of analysis and logical deduction, the distinction between cause and effect, clearness and accuracy in thought and statement, should always be important objects for every teacher. Reasoning power is so complex that its measurement is exceedingly difficult. The large tests made in New York and in Boston show that reasoning power is independent of skill in the fundamental operations. A group of investigators in the fields of geometry or analysis would not compare in arithmetical accuracy and speed with the work of trained computors.

5. There are undoubtedly many valuable factors in the teaching of arithmetic which are not susceptible of distinct separation and numerical tabulation. Other factors, though probably measurable, are complex and involved. Hence it is desirable to examine the simplest mechanical processes first.

6. In accordance with the above suggestions, we recommend that individual teachers and school systems give tests which have had wide application and compare their results with the widely established standards, noting the agreements and differences. The latter should be explained if possible. The results, including explanations of anomalies and suggestions arising from the local test, should be communicated to the investigators whose standards were used.

7. Conclusions should not be drawn from a small number of observations, neither should statistics be selected to prove a preconceived idea. Questions arising from our experience may arouse fruitful investigations, but it is just as important to disprove an erroneous assumption as it is to confirm our suspicions. We feel that extended and unwarranted deductions from the

simple tests in the fundamentals of arithmetic will tend to defeat their primary purpose of improving school conditions and will create a strong opposition to them, largely based on the grounds which resulted in the overthrow of the severe examination system formerly prevalent in our schools.

8. In conclusion, we feel that the tests already given by Courtis and others have clearly detected grave technical weakness in many schools in the fundamentals of arithmetic, and that such carefully devised tests should be used for this purpose. When such weakness is detected, remedial measures should be instituted to avoid the complex situation proved by recent investigations to exist in New York, Boston, and elsewhere. Scientific methods and effective teaching are impossible when large classes comprise many grades of arithmetic ability in the fundamental processes. This is the situation now confronting our teachers and we urgently recommend this association to use its influence in correcting this glaring defect in the supervision and organization of our schools.

Respectfully submitted,

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